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The Microstrip Open-Ring Resonator

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***Abstract*—The open-ring microstrip resonator is analyzed by utilizing the two-dimensional magnetic wall model. The solution and the numerical results for the eigenvalues and the electromagnetic fields for various resonant modes are presented. It is shown that the experimental results are in good agreement with the theoretical predictions based on this model.**

I. Introduction

OPEN-RING microstrip resonators have been proposed for applications in microwave filters [1] and as planar antenna elements [2]. The structure is analyzed in this paper by utilizing the two-dimensional magnetic wall model, and the results computed for eigenvalues (resonant frequencies) and electromagnetic field distribution are presented. The problem is similar to that of the disc and the closed-ring resonators, which have been studied extensively in recent years for applications as resonators and planar antenna elements [2]–[10]. The magnetic wall model, though an approximate one, has been successfully used in the past for many microstrip patch geometries, including discs and annular rings.

II. The Model and the Eigenvalue Problem for the Microstrip Open-Ring Resonator

The microstrip open-ring resonator, as shown in Fig. 1(a), consists of a planar ring segment having an inner radius r_i, an outer radius r_a, and an angle α of the open segment on a substrate of height h. The corresponding magnetic wall model for the resonator is defined in Fig. 1(b). This consists of a ring with effective inner and outer radii $r_{i,\mathrm{eff}}$ and $r_{a,\mathrm{eff}}$, respectively, vertical magnetic walls

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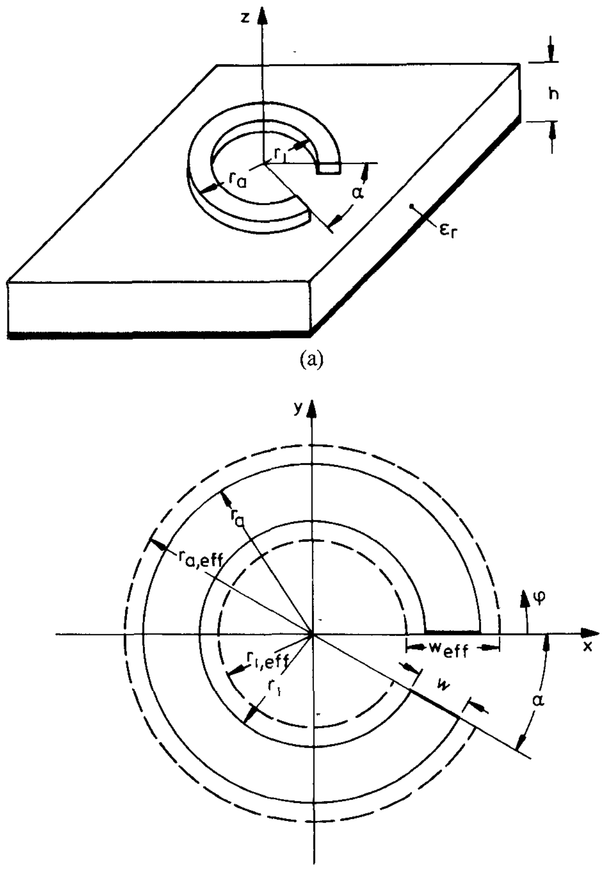

Fig. 1. (a) The microstrip open-ring resonator. (b) The magnetic wall model used for formulating the eigenvalue problem.

between the edges and the ground planes, and the resonator filled with a medium having a relative dielectric constant ϵ_{eff} corresponding to that of the microstrip line. The effective radii are given by

$$r_{i,\text{eff}} = r_i - \Delta r_i, \qquad \Delta r_i = \frac{w_{\text{eff}} - w}{2} \frac{r_i}{r_a}$$

$$r_{a,\text{eff}} = r_a + \Delta r_a, \qquad \Delta r_a = \frac{w_{\text{eff}} - w}{2}. \tag{1}$$

Here, w_{eff} is the effective width of the microstrip line, and can be determined by using various microstrip line models including that of Kompa and Mehran [11]. The factor r_i/r_a in the equation for Δr_i has been introduced to describe the vanishing stray field in the limiting case $r_i \to 0$ [6]. No attempt has been made to include the effects of stray fields in the gap of the open-ring since the electromagnetic fields in this region are strongly dependent on the field structure of various modes. For certain modes, e.g., the lowest order mode, the effect of the gap can be approximated by introducing a gap capacitance and an effective angle to account for the fringing stray fields at the ends. In general, however, the influence of the stray fields in the gap can only be described accurately by formulating the complete boundary value problem.

The dispersion effects can, if desired, be included in the model by utilizing the frequency-dependent dynamic dielectric constant ϵ_{dyn} as shown for the case of discs and rings [3], [6] and are not significant, as long as the width of the microstrip line forming the open-ring resonator is small.

Since the height of the substrate material h is assumed to be small compared to the wavelength on the microstrip ring, only modes with electromagnetic fields, which are independent of the z-coordinate (Fig. 1) are considered. Using the assumptions made above, the expressions for the electromagnetic fields are found to be

$$E_z = A\left\{ J_\nu(kr) - \frac{J_\nu'(kr_{a,\text{eff}})}{Y_\nu'(kr_{a,\text{eff}})} Y_\nu(kr) \right\} \cos(\nu\phi) \tag{2}$$

$$H_r = \frac{\nu}{j\omega\mu_0 r} A\left\{ J_\nu(kr) - \frac{J_\nu'(kr_{a,\text{eff}})}{Y_\nu'(kr_{a,\text{eff}})} Y_\nu(kr) \right\} \sin(\nu\phi) \tag{3}$$

$$H_\phi = \frac{k}{j\omega\mu_0} A\left\{ J_\nu'(kr) - \frac{J_\nu'(kr_{a,\text{eff}})}{Y_\nu'(kr_{a,\text{eff}})} Y_\nu'(kr) \right\} \cos(\nu\phi) \tag{4}$$

where

$k = \omega\sqrt{\epsilon_0 \epsilon_{\text{eff}} \mu_0}$ is the wavenumber and ν is given by

$$\nu = \frac{m\pi}{2\pi - \alpha} \tag{5}$$

such that the boundary conditions for the tangential H_r-component at $\phi = 0$ and $\phi = -\alpha$, i.e., at the ends of the ring segment (Fig. 1), are satisfied. From the boundary conditions for the H_ϕ-component, which is tangential to the cylindrical boundary of the model resonator at $r = r_{i,\text{eff}}$ and $r = r_{a,\text{eff}}$, the eigenvalue equation is found to be

$$\frac{J_\nu'(kr_{a,\text{eff}})}{Y_\nu'(kr_{a,\text{eff}})} - \frac{J_\nu'(kr_{i,\text{eff}})}{Y_\nu'(kr_{i,\text{eff}})} = 0. \tag{6}$$

In the above expressions, J_ν is the cylindrical function of the first kind (Bessel function), Y_ν is the cylindrical function of the second kind (Neumann function) of order ν, which is dependent on the gap angle α and is, in general, not an integer value. The primes represent the derivative of the functions with respect to the total argument.

III. The Modes of the Open-Ring Resonator

The electromagnetic fields associated with all the $E_{mn0}(TM_{mn0})$ modes with respect to the z-axis can be evaluated from (2) though (6). Here m describes the dependence of the electromagnetic field components on the azimuthal angle and n is the order of the root of the eigenvalue equation (6), and describes the radial behavior. The resulting three different classes of modes are discussed below.

A. The $E_{0n0}(TM_{0n0})$ Modes

If $m = 0$, the electromagnetic fields are independent of the azimuthal angle ϕ. Fig. 2 shows the field lines for the magnetic field strength of the $E_{010}(TM_{010})$ mode in the open-ring resonator with a gap angle $\alpha = 5°$. The field lines are open circles which begin and end on the magnetic walls

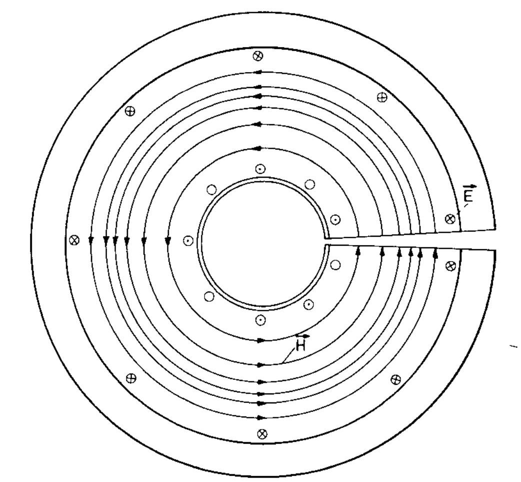

Fig. 2. Field distribution of the electromagnetic field of the E_{0n0} mode. Gap angle $\alpha = 5°$. The figure shows the real geometrical structure of the resonator (inner circles) and the geometry of the magnetic wall model (outer circles).

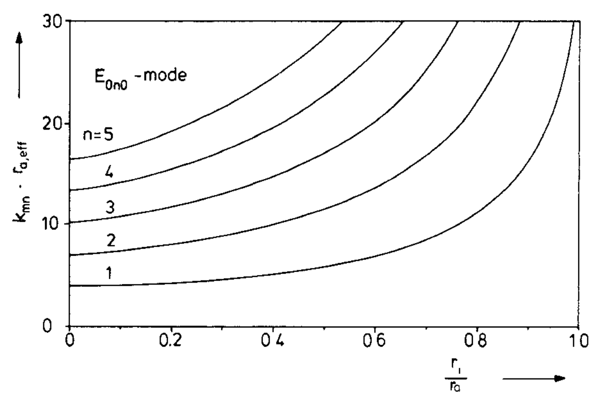

Fig. 3. The eigenvalues of the E_{0n0} modes for $n = 1$ to $n = 4$ versus r_i/r_a.

of the curved microstrip-line model. The electrical field is maximum at the magnetic walls ($r = r_{i,\text{eff}}$ and $r = r_{a,\text{eff}}$) and has a zero near the middle of the microstrip line. The exact position of the zero depends on the curvature and width of the microstrip line. The magnetic field strength H_ϕ has a maximum value at the position of the zero of E_z. Fig. 3 shows the eigenvalues $k_{mn}r_{a,\text{eff}}$ as a function of the quotient r_i/r_a for different values of n. It should be noted that the eigenvalues of the E_{0n0} modes are dependent on the inner and the outer radius of the open-ring resonator only, and not on the gap angle α. This property may be easily understood if the field distribution as shown in Fig. 2 is considered. The geometrical form of the electromagnetic field distribution is not changed by varying the gap angle α. The eigenvalue equation and the eigenvalues of the E_{0n0} modes of the open-ring resonator are exactly the same as those for the E_{0n0} modes of the closed-ring resonator. The E_{0n0} modes normally are not used in practical applications because the eigenvalues become very large for small line widths (Fig. 3).

B. The $E_{m10}(TM_{m10})$ Modes

The E_{110} mode is the one with the lowest eigenvalue, and, therefore, is of interest for practical applications. As Fig. 4 shows, the E_{110} mode can be considered as the resonance on a curved $\lambda/2$ microstrip line. The field distribution of the magnetic field strength far away from the gap is nearly equal to that of a straight microstrip line. Only in the gap area does the field distribution change, because the field lines must be perpendicular to the boundaries at the end of the lines. As the gap angle approaches zero, the field distribution does not converge into that of the closed-ring resonator because the magnetic walls at the ends of the ring line do not vanish and, as shown in Fig. 5, the eigenvalues of the E_{110} mode are nearly half the value of those for the closed-ring resonator. The eigenvalues of all the E_{m10} modes, as should be expected, are dependent on the gap angle α. Fig. 6 shows the influence of the gap angle on the radial distribution of the electric field component E_z. For small gap angles (e.g., 5°), the electric field component E_z (considered at the azimuthal angle ϕ where it is maximum) is nearly constant across the microstrip line and only changes slightly from the inner to the outer circumference, whereas for $\alpha = 180°$, the variation in the magnitude of E_z is more significant as one goes from the inner to the outer circumference.

If m is changed from 1 to 2, an additional zero of the azimuthal component H_ϕ is found and the corresponding field distribution is shown in Fig. 4(b). The mean value of the resonator length now is nearly one line wavelength. The E_z-components of the modes with higher m-values are more strongly dependent on the radial coordinate than that of the E_{110} mode.

C. The $E_{mn0}(TM_{mn0})$ Modes

For values of n higher than 1, i.e., for a higher order solution of the eigenvalue equation (6), the E_z-component and simultaneously the H_r-component (see (2) and (3)) have one or more zeros as a function of the radial coordinate. Therefore, the magnetic field lines have a distribution as shown in Fig. 7, where a concentration of the field lines near the gap can be recognized. Because of the strong dependence of the field components on the radial coordinate, the eigenvalues of the E_{mn0} modes with values of n higher than 1 become very large for ring resonators with a small line width. This is similar to the case of the E_{0n0} modes, and, therefore, these modes normally are also not of great interest for practical applications.

IV. Measurements

More than 150 resonance frequencies of modes on different open-ring resonators have been measured and compared with the theoretical results. The experimental resonators were built on RT-Duroid substrate material ($\epsilon_r = 2.23$), the height of the substrate material was $h = 0.79$ mm, and the ring resonators had a mean value of the radius of $R = 16$ mm. Resonators of widths $w = 4$, 6, 10, 16, and 24 mm and gap angles α ranging from 1 to 20° have been studied. In each case, the first seven resonances of the resonance spectrum were measured. Fig. 8 shows the comparison between some of the experimental results ($\times$) with the theoretical results (——) as a function of the width of the ring resonator. As seen from these figures, the agree-

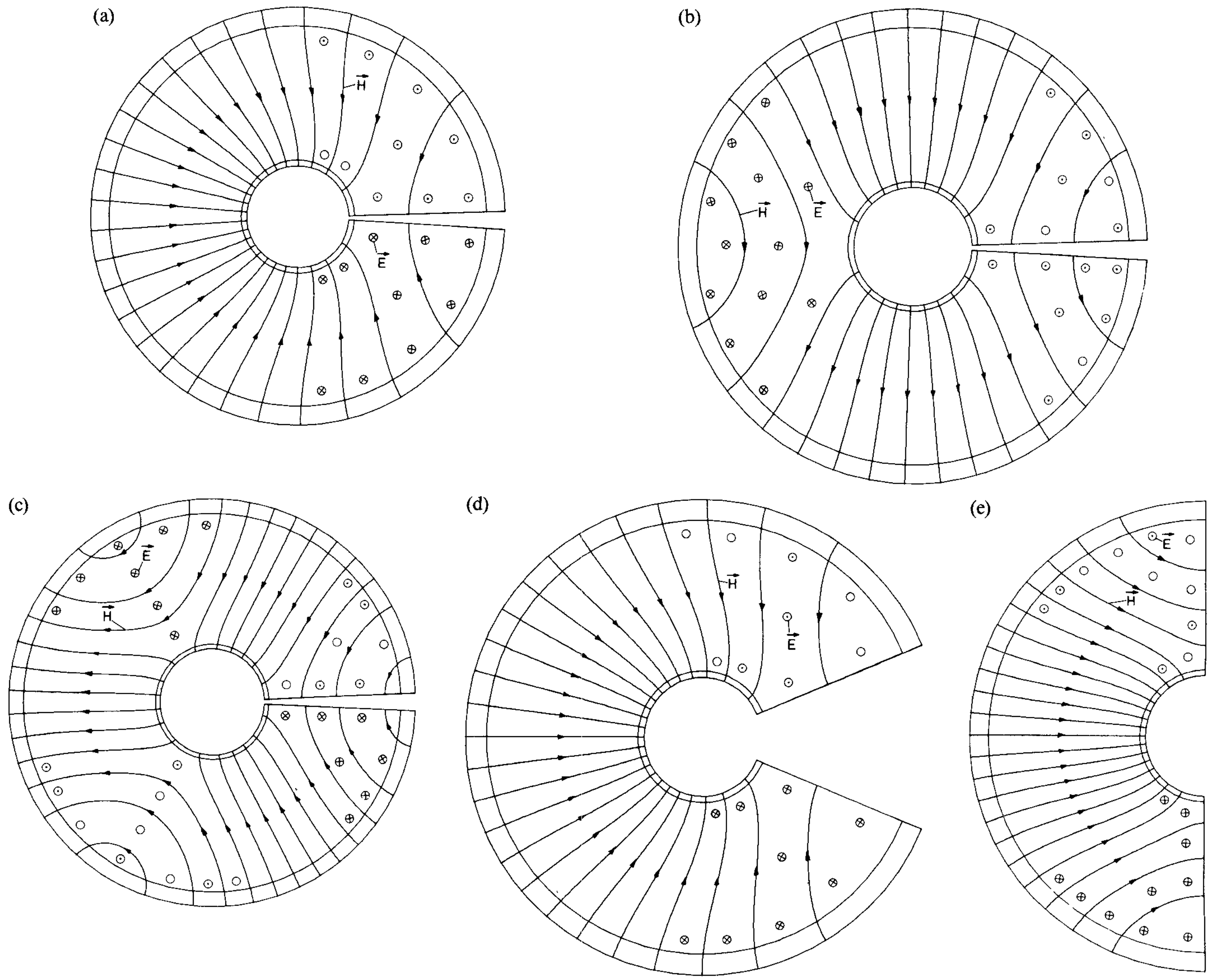

Fig. 4. Field distribution of the electromagnetic field of the E_{m10} modes for (a) $m=1$, (b) $m=2$ and (c) $m=3$ for an open-ring resonator of gap angle $\alpha=5°$ and field distribution of the E_{110} mode for an open-ring resonator with a gap angle (d) $\alpha=45°$ and (e) $\alpha=180°$.

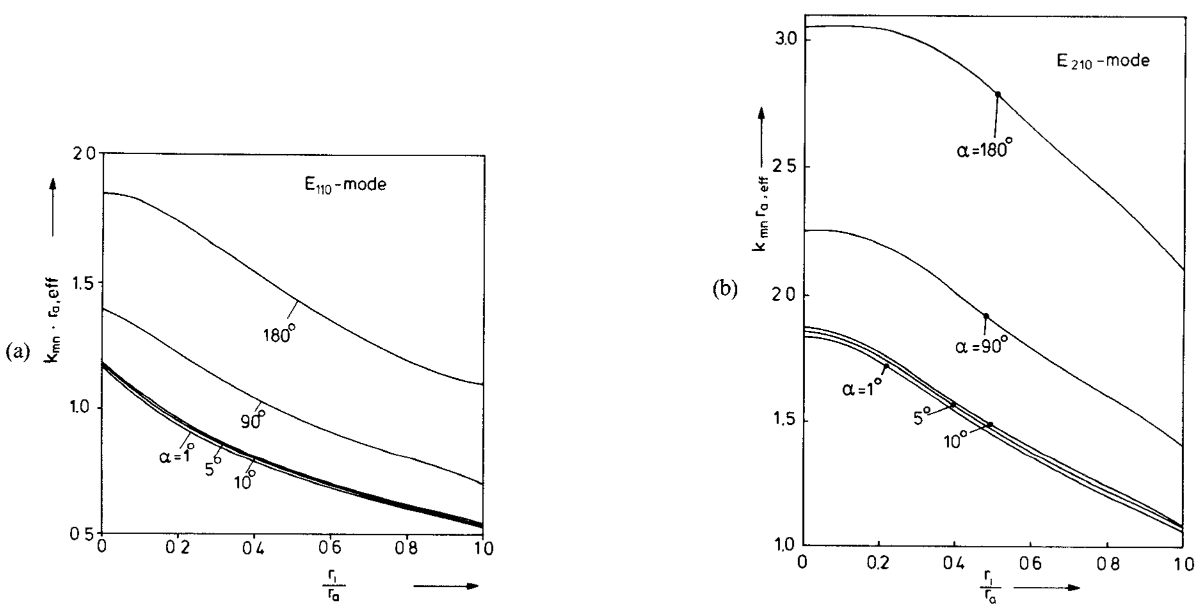

Fig. 5. The eigenvalues of the E_{110} mode and the E_{210} mode as a function of r_i/r_a for different gap angles.

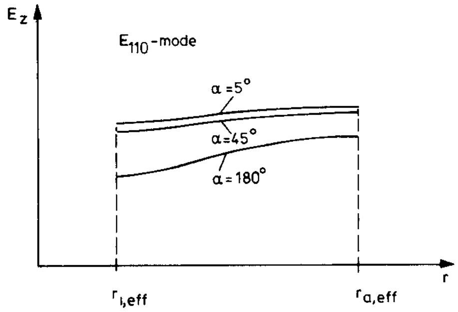

Fig. 6. Dependence of the electrical-field component E_z of the E_{110} mode on the radial coordinate for three different values of the gap angle α. $r_i/r_a = 0.3$, $r_a = 1$ cm, substrate material: RT/Duroid ($\epsilon_r = 2.23$, $h = 0.79$ cm).

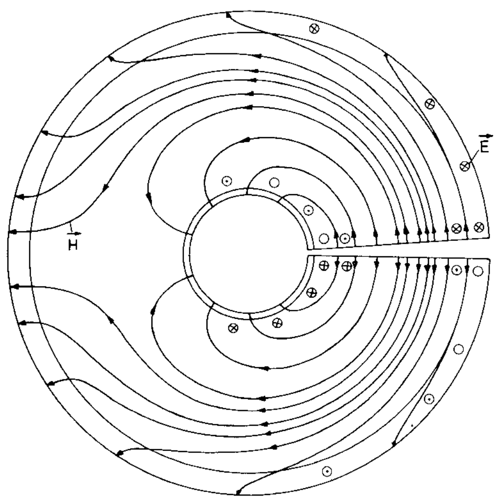

Fig. 7. Field distribution of the electromagnetic field of the E_{120} mode. Gap angle $\alpha = 5°$.

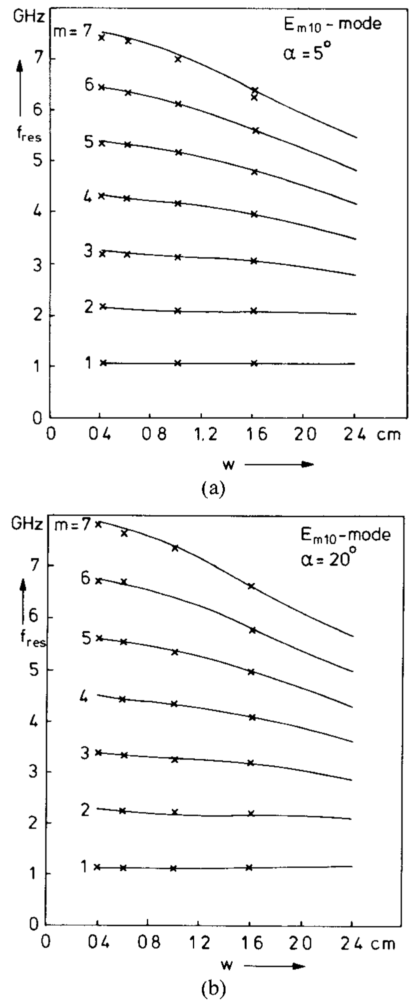

Fig. 8. Measured (×) and calculated (——) resonant frequencies of open-ring resonators; E_{m10} modes for $m = 1$ to $m = 7$ for a gap angle of (a) $\alpha = 5°$ and (b) $\alpha = 20°$ as a function of the line width w. Mean radius of the ring resonator: $R = 16$ mm. Substrate material: RT/Duroid ($\epsilon_r = 2.23$, $h = 0.79$ cm).

TABLE IA
NORMALIZED DIFFERENCE BETWEEN THE MEASURED AND THE CALCULATED RESONANCE FREQUENCIES FOR AN OPEN-RING RESONATOR WITH GAP ANGLE $\alpha = 1°$

m | w/cm	0.4	0.6	1.0	1.6	2.4
1	-4.6%	-4.4%	-4.0%	-2.9%	-0.5%
2	-2.5%	-2.3%	-1.9%	-1.7%	-2.3%
3	-4.6%	-4.5%	-4.5%	-1.2%	-3.6%
4	-2.9%	-2.8%	-2.6%	-3.3%	-3.5%
5	-4.8%	-5.0%	+5.0%	-4.5%	-4.6%
6	-3.0%	-3.2%	-3.3%	-3.7%	-2.4%
7	-4.9%	-6.0%	-5.2%	-6.8%	-4.4%

TABLE IB
NORMALIZED DIFFERENCE BETWEEN THE MEASURED AND THE CALCULATED RESONANCE FREQUENCIES FOR $\alpha = 15°$

m | w/cm	0.4	0.6	1.0	1.6
1	-0.3%	-0.2%	-1.1%	-1.4%
2	-0.4%	+0.1%	+1.7%	+5.5%
3	-0.8%	-0.2%	+0.5%	+0.3%
4	-0.7%	-0.4%	+0.4%	-0.1%
5	-0.7%	-0.2%	-1.7%	-0.7%
6	-0.7%	-0.5%	-0.3%	-0.7%
7	-1.1%	-1.0%	-0.6%	-0.6%

ment between theory and experiment is quite good, despite the fact that the influence of the end effects of the open microstrip ring resonator has not been taken into account.

In order to demonstrate clearly as to how large the error due to the ideal gap assumption is, Table I additionally shows the difference between the measured resonant frequencies and the calculated resonant frequencies for different open-ring resonators in this case with a) a gap angle of $\alpha = 1°$ and b) $\alpha = 15°$. The data presented in Table I demonstrate that the influence of the gap capacitance on the resonant frequency is much larger for the case of the small gap angle ($\alpha = 1°$) than for the case of the large gap angle ($\alpha = 15°$). Additionally, Table IA shows that, as should be expected from the field distribution, the influence of the odd-mode gap capacitance ($m = 1, 3, \cdots$) on the resonant frequency is much larger than the influence of the even-mode gap capacitance ($m = 2, 4, 6, \cdots$). This also suggests that the open-ring resonator could be used to measure the even-mode and odd-mode gap capacitances between two microstrip lines. Table IB shows that, for large gap angles, the influence of the end effect on the resonant frequency becomes small. The difference between the measured and the calculated results is of the order of 1 percent or smaller for this case of a 15° gap angle.

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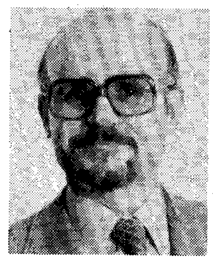

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